School Finance, Peers, and Choice

William Phan[1] Ryan Tierney[2] Yu Zhou[3]

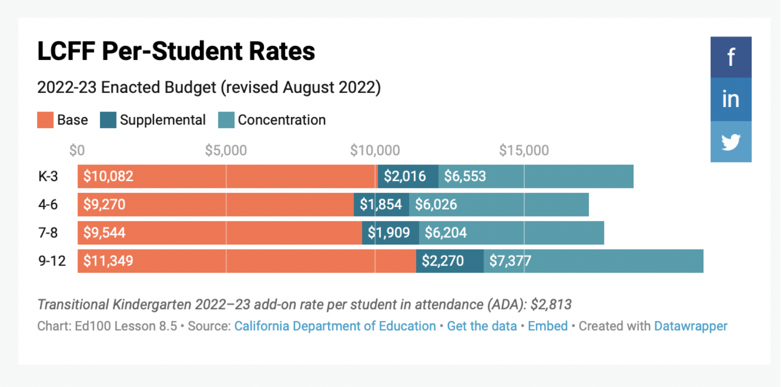
[1]North Carolina State University

[2]University of Southern Denmark

[3]Nagoya University

Turun Yliopisto, March 2026

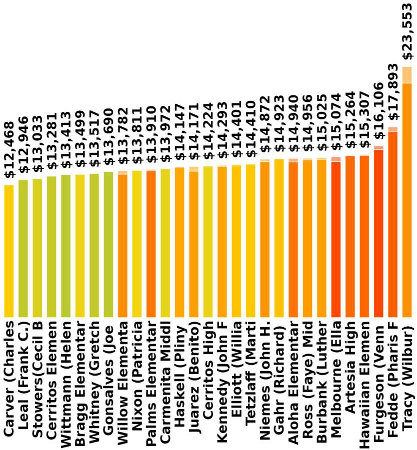
California Education Funding Formula



Funding

- California: \$7.5 billion (10% of LCFF)
- North Carolina: \$1.6 billion (7%)
- Texas: \$3.4 billion (6%)

Funding



California ABC Unified School District 2020-2021

Funding

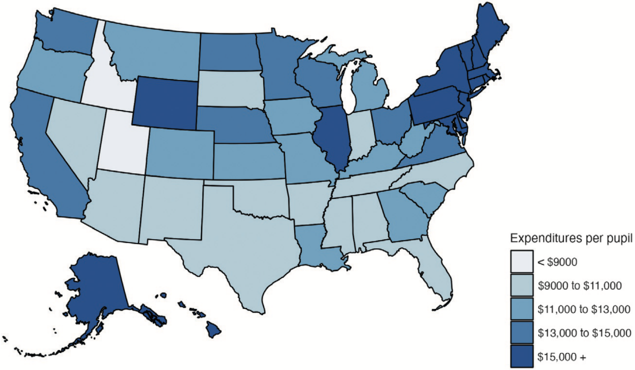


FIG. 3

Per-pupil expenditure by state, 2019.

Source*: NCES 2021 digest (https://nces.ed.gov/programs/digest/2021menu_tables.asp), table 236.65.*

Funding

Table 1 Distribution of funding source makeup with representative states, 2019 (percent)

Funding source	Mean	Minimum	Maximum
Local	42.26	2.10 (Hawaii)	91.97 (DC)
State	50.07	26.57 (Illinois)	90.29 (Vermont)
Federal	8.63	4.12 (New Jersey)	15.44 (Alaska)

Source*: NCES 2021 digest (https://nces.ed.gov/programs/digest/2021menu_tables.asp), table 235.20.*

Literature

School Finance
$ vs. Outcomes

Sorting & Peer Effects
Neighbors, Peers vs. Preferences or Outcomes

School Choice
Designing assignment mechanisms

Literature

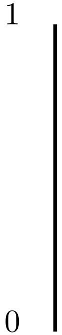
School Finance
Handel and Hanushek [2023], Fiszbein and Schady [2009], Angrist, Bettinger, Bloom, King, and Kremer [2002], Benhassine, Devoto, Duflo, Dupas, and Pouliquen [2015], Neilson [2021], Bobba, Ederer, Leon-Ciliotta, Neilson, and Nieddu [2021]

Sorting & Peer Effects
Epple and Romano [1998], Hoxby [2001, 2003], Rothstein [2006], Abdulkadiroğlu, Angrist, and Pathak [2014], Abdulkadiroğlu, Pathak, Schellenberg, and Walters [2020], Avery and Pathak [2021]

School Choice
Abdulkadiroğlu and Sönmez [2003], Abdulkadiroğlu, Pathak, and Roth [2009], Leshno [2022]

Contribution

1. A *resource* model of school choice.
2. Propose *enrollment-based funding equilibria*.
3. Applications
   3.1 Funding in schools
   3.2 Peer effects
   3.3 Vouchers
4. New proof techniques
   4.1 crowding and funding drive the difficulties
   4.2 not Tarski fixed point
   4.3 not Decomposition Lemma


1
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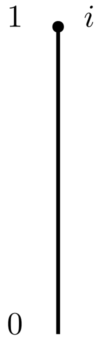
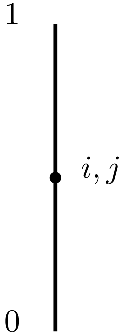


1
i
0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i, j
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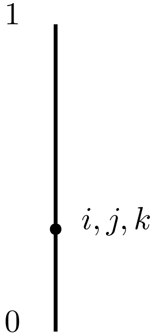
1
i, j, k
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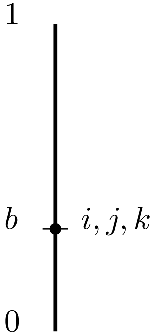
1
b
i, j, k
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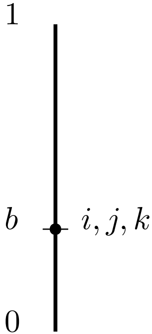
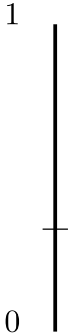


$$\frac{1}{b} = capacity$$



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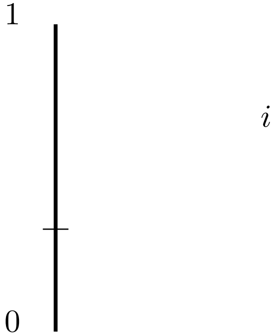
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i
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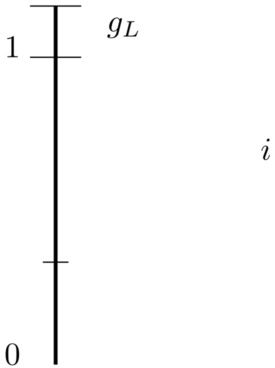
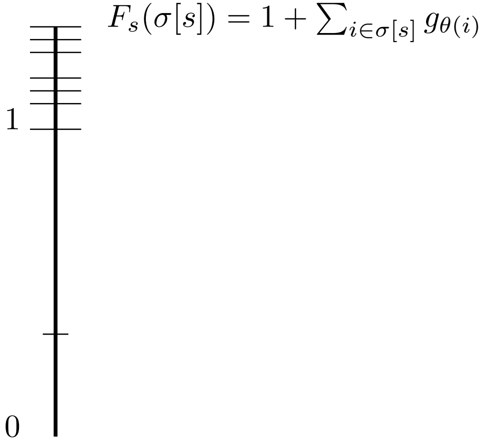


g_L
1
i
0

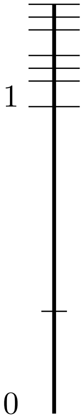
California: general staff hiring, general teacher raises.

North Carolina: general student resources.

Texas: half of the Compensatory Allotment can be used for general spending.

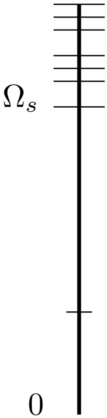

$F_s(\sigma[s]) = 1 + \sum_{i \in \sigma[s]} g_{\theta(i)}$
1
0

$$F_s(\sigma[s]) = 1 + \sum_{i\in\sigma[s]} g_{\theta(i)}$$

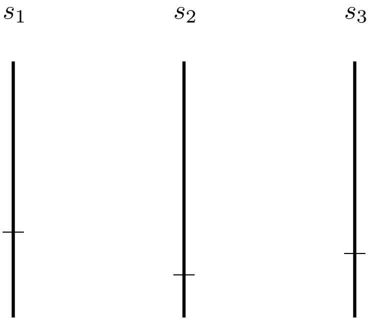


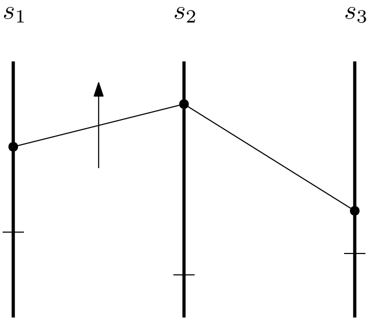
$$\frac{F_s(\sigma[s])}{b_s}$$

$$F_s(\sigma[s]) = \Omega_s + \sum_{i \in \sigma[s]} g_{\theta(i)}$$



$$\frac{F_s(\sigma[s])}{b_s}$$


s_1
s_2
s_3


s_1
s_2
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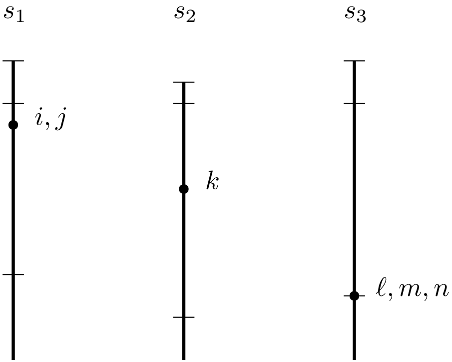
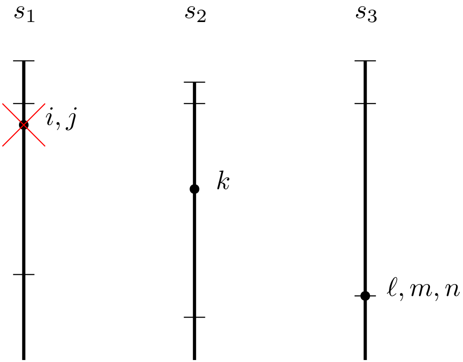


s_1
s_2
s_3
i, j
k
ℓ, m, n



s_1
s_2
s_3
i, j
k
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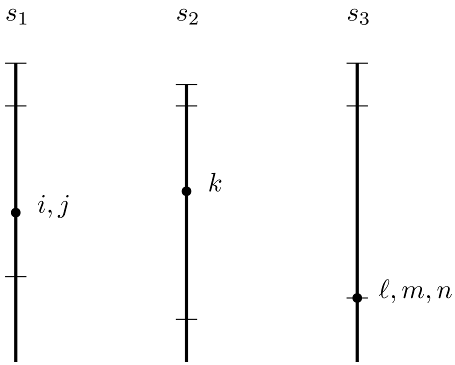

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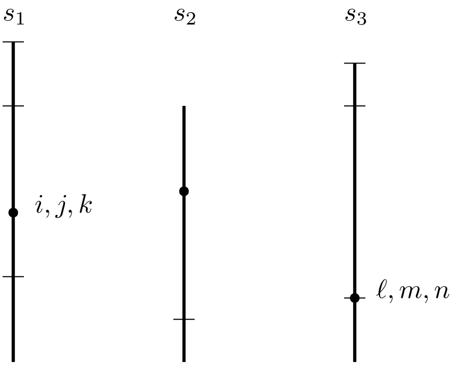


s_1
s_2
s_3
i, j, k
ℓ, m, n

A **student's externality** has 3 parts:

1. negative crowding effect
2. positive funding
3. negative/positive personal peer effect

Model

L, H	low and high grant students
S	schools
$\mathbb{R}_+ \times S$	consumption space of students
R_i	preference relation of student i
Ω_s	initial capital of school s
b_s	minimum distribution of school s
g_L	low grant
g_H	high grant
$\succ_s$	priority order of school s over students
$(L, H, S, \Omega, b, g, \succ, R)$	**school choice with funding problem**
ρ_s	distribution of school s
$\sigma(i)$	match of student i
$(\boldsymbol{\rho}, \sigma)$	allocation

Model

Total Funding

$$F_s(\sigma) = \Omega_s + \sum_{i \in \sigma[s]} g_{\theta(i)}$$

An **allocation** $(\boldsymbol{\rho}, \sigma)$ is such that for each school s,

1. (Distribution Feasibility)

$$\rho_s \cdot |\sigma[s]| \leq F_s(\sigma).$$

2. (Minimum Bound)

$$\rho_s \geq b_s.$$

Axiom

Fairness

No Envy at Interior + No Justified Envy at Boundary

Axiom

Fairness
If student i prefers school s to their match at the current level, then s is

1. full, and
2. all students at s have higher priority.

Axiom

Fairness
If student i prefers school s to their match at the current level, then s is

1. is at minimum b_s, and
2. all students at s have higher priority.

Axiom

Fairness:

$$(\boldsymbol{\rho}, s)\ P_i\ (\boldsymbol{\rho}, \sigma(i)) \quad \Rightarrow \quad \begin{array}{l} 1.\ \rho_s = b_s, \text{ and} \\ 2.\ \text{for each } j \in \sigma[s],\ j \succ_s i, \end{array}$$

Axiom

Fairness:

$$(\boldsymbol{\rho}, s)\ P_i\ (\boldsymbol{\rho}, \sigma(i)) \quad \Rightarrow \quad \begin{array}{l} 1.\ \rho_s = b_s, \text{ and} \\ 2 \text{ for each } j \in \sigma[s], \\ \quad \text{a. } j \succ_s i, \text{ or} \\ \quad \text{b. } (\sigma[s] \setminus \{i\}) \cup \{j\} \text{ is not feasible.} \end{array}$$

Enrollment-Based Funding Equilibrium (EBFE)

An EBFE is an allocation $(\boldsymbol{\rho}, \sigma)$ such that:

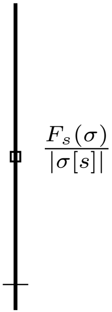
1. (IR + Fairness) $(\boldsymbol{\rho}, \sigma)$ is *individually rational* and *fair*.
2. (Exhaustive) For each school s with $\sigma[s] \neq \emptyset$,

$$\left\lfloor \frac{F_s(\sigma)}{\rho_s} \right\rfloor = |\sigma[s]|.$$

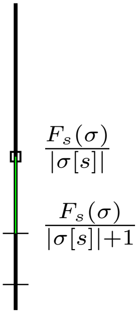
3. (Inferior Empty Schools) For each school s with $\sigma[s] = \emptyset$, $\rho_s = \Omega_s$, and for each $i \in N$,

$$(\boldsymbol{\rho}, \sigma(i)) \; P_i \; (\boldsymbol{\rho}, s).$$

Exhaustiveness Discrepancy



Exhaustiveness Discrepancy



EBFE Is a Walrasian Concept

An EBFE is an allocation $(\boldsymbol{\rho}, \sigma)$ such that:

1. (Fairness) Consumer maximization.
2. (Exhaustive) Market clearing with one seat error.
3. (Inferior Empty Schools) Set price of unassigned good to zero.

Price Equilibrium with Rigidities

Drèze [1975], Talman and Yang [2008], Andersson and Svensson [2014], Herings [2018].

Existence

A preference is **linear** if $(r, s)\ R_i\ (g, t)$ implies that there is valuation function $v_i : S \to \mathbb{R}_+$, such that

$$(r, s)\ R_i\ (g, t) \iff rv_i(s) \geq gv_i(t).$$

Main Results: Existence & Structural Properties

Theorem 1 On the NCBI linear domain, EBFE exist.

Proposition 1 The set of *fair* allocations forms an upper lattice.

Theorem 2 Given a profile from the NCBI linear domain, there is a student-optimal EBFE.

Main Results: Existence & Structural Properties

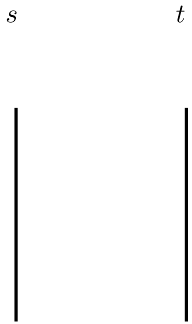
Proof Intuition:

- *Fair* allocations trivially exist.
- Prove upper lattice of *fair* set, so there is a student-optimal *fair* allocation.
- This allocation may fail exhaustiveness, so we must augment the matching to find an EBFE.

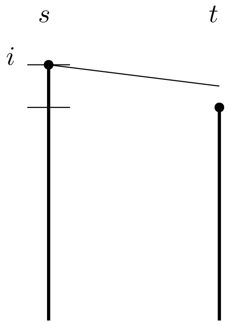
Main Results: Some Things Break

- Set of distributions that are part of some EBFE do not form a lattice.
- Exists *fair* that are not EBFE.
- No Rural Hospital / Lone Wolf theorems.
- Manipulability of student-optimal EBFE mechanism.

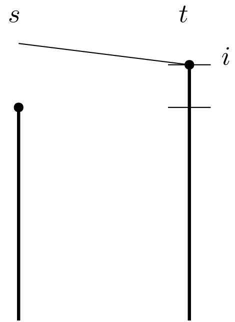
No Rural Hospital Theorem



No Rural Hospital Theorem



No Rural Hospital Theorem



Impossibility

Proposition 3 A mechanism that is a selection from the student-optimal EBFE correspondence is not *strategy-proof*.

General Model

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General Model

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g_i	total externality of i
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S	schools
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Ω_s	initial capital of school s
b_s	minimum distribution of school s
g_i	total externality of i
v_i	**voucher of $\boldsymbol{i}$**
$\succ_s$	priority order of school s over students
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General Model

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S	schools
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S	schools
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v_i	voucher of i
Ch_s	preferences of school s over students
$(I, S, Ch, \Omega, b, g, v, R)$	**school choice with funding problem**

School Choice Functions

The choice function $Ch_s(A) \subseteq A$ of each school s satisfies:

- Substitutability
  $B \supseteq A \implies Re_s(B) \supseteq Re_s(A)$
- Independence of Rejected Students
  $A' \subseteq Re_s(A) \implies Ch_s(A \setminus A') = Ch_s(A)$
- Budget Constraint
  $b_s|Ch_s(A)| \leq F_s(Ch_s(A))$

Fair Lattice

Proposition 1 (Fair Upper Lattice) Consider two *fair* allocations $(\boldsymbol{\rho}, \sigma)$ and $(\boldsymbol{\gamma}, \tau)$ for a preference profile from the NCBI linear domain. Let $\boldsymbol{\zeta} = \boldsymbol{\rho} \vee \boldsymbol{\gamma}$. There is a matching μ such that $(\boldsymbol{\zeta}, \mu)$ is a *fair* allocation, and moreover, for each $i \in N$,

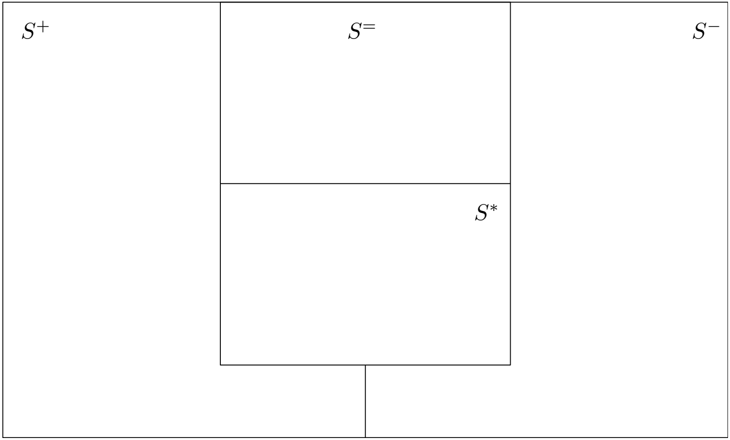
$$(\boldsymbol{\zeta}, \mu(i)) \; R_i \; \max_{R_i}\{(\boldsymbol{\rho}, \sigma(i)), (\boldsymbol{\gamma}, \tau(i))\}.$$

Fair Lattice

Sketch Proof:

1. Pick two fair allocations.
2. Define Transfer graph.
3. Implement weakly welfare-increasing cycles.
4. (Key) Make sure you maintain feasibility.

Fair Lattice



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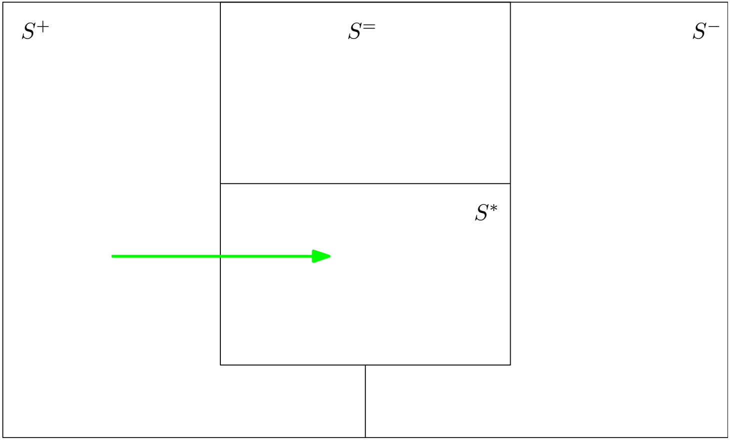

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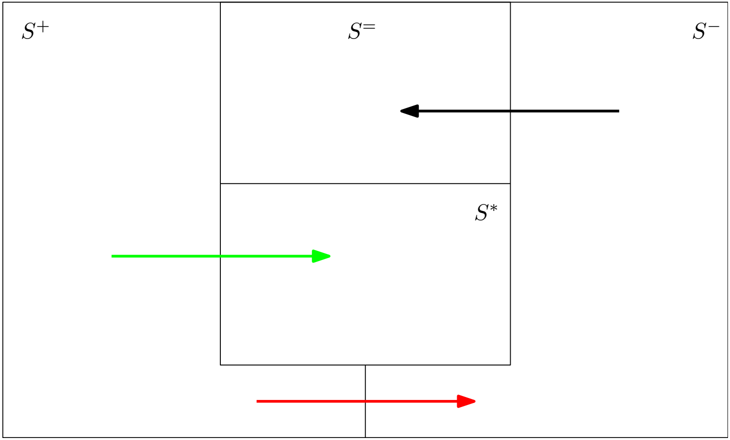

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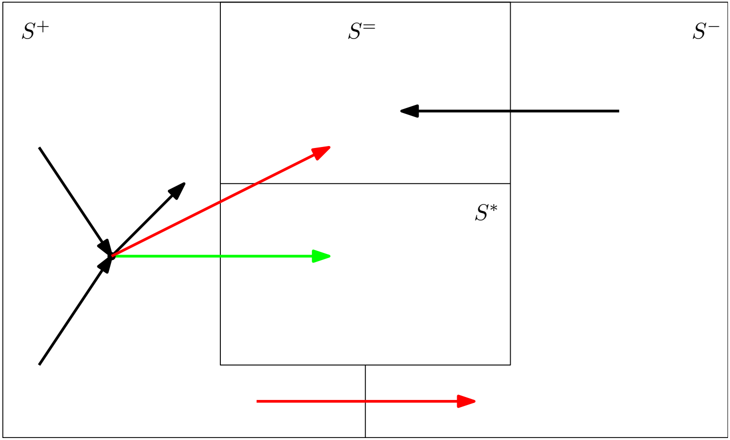

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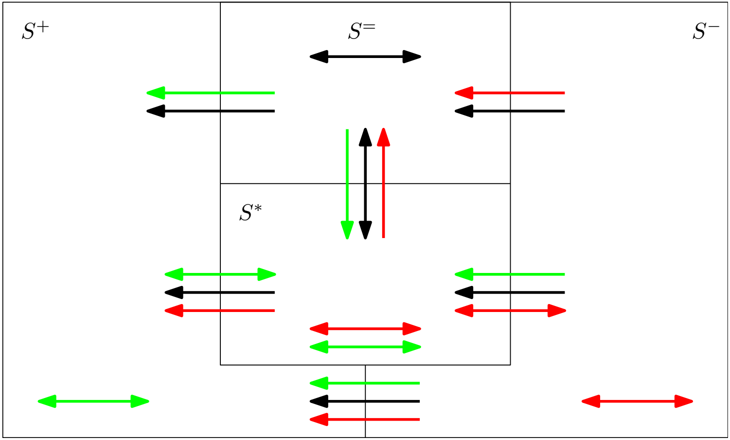
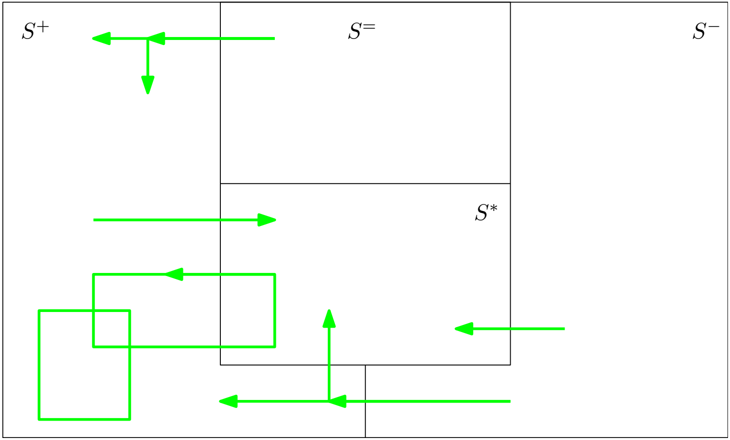
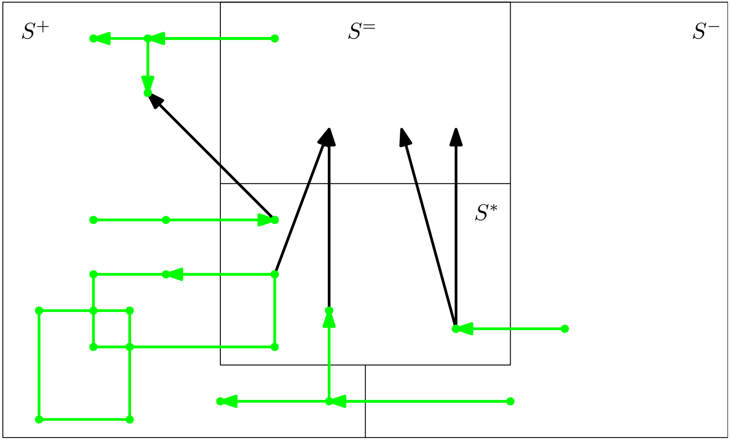


Fair Lattice



Fair Lattice



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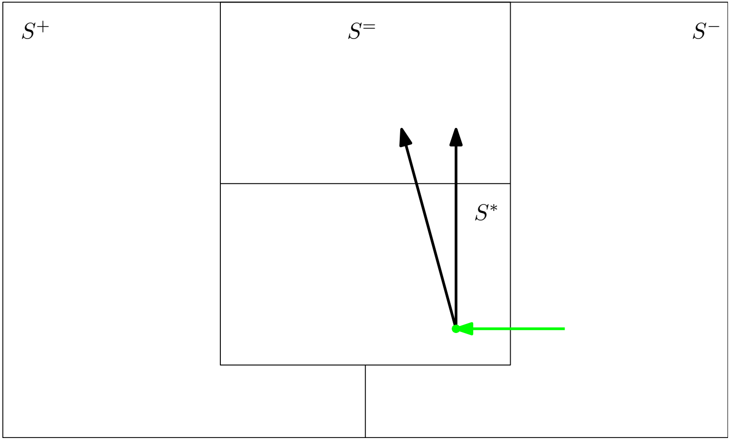

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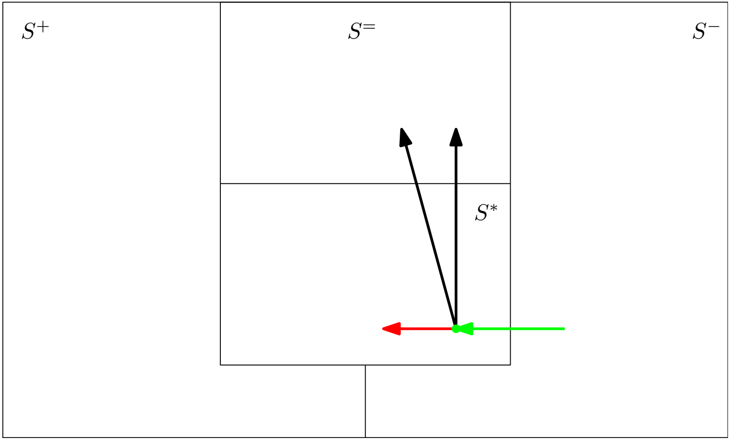

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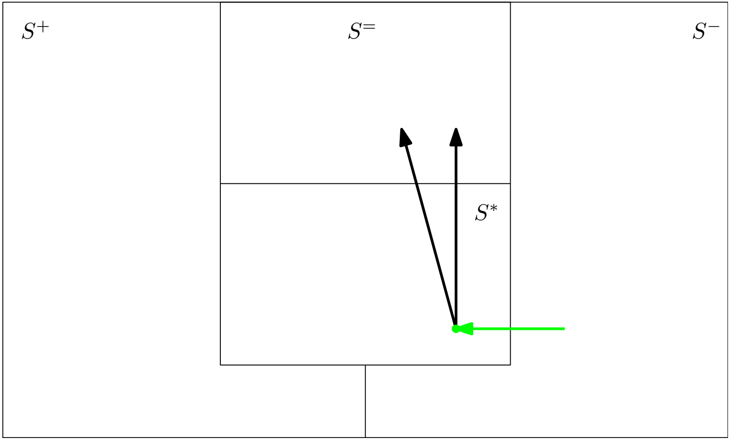

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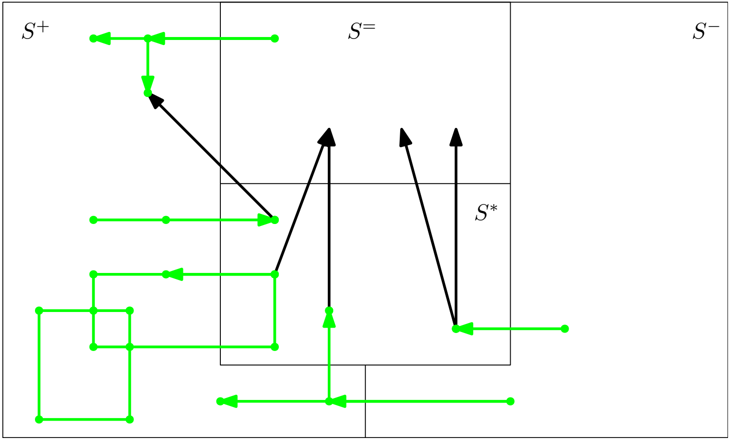

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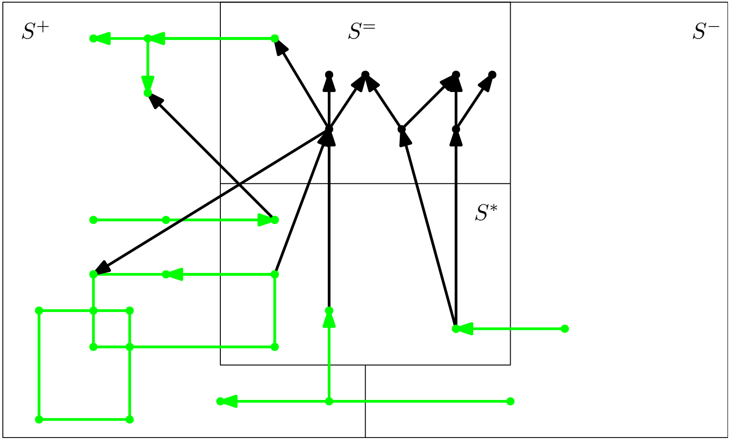

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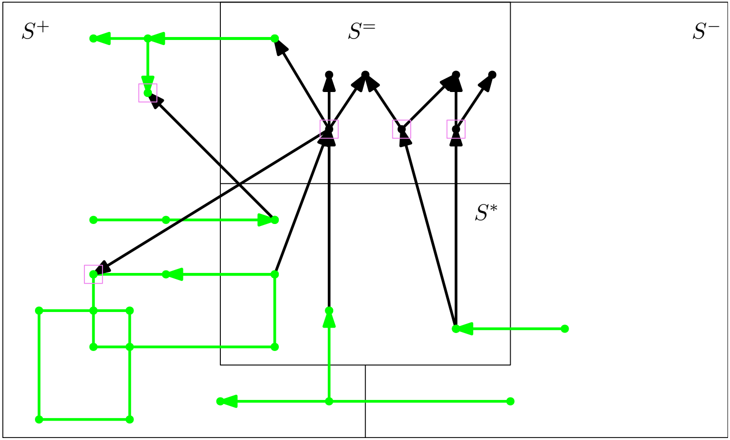

Conclusion

Thank you!

School Finance

Sorting & Peer Effects

School Choice

Literature

School Choice
Abdulkadiroğlu and Sönmez [2003], Phan et al. [2024].

Matching with Externalities
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