Equal Opportunity in School Choice

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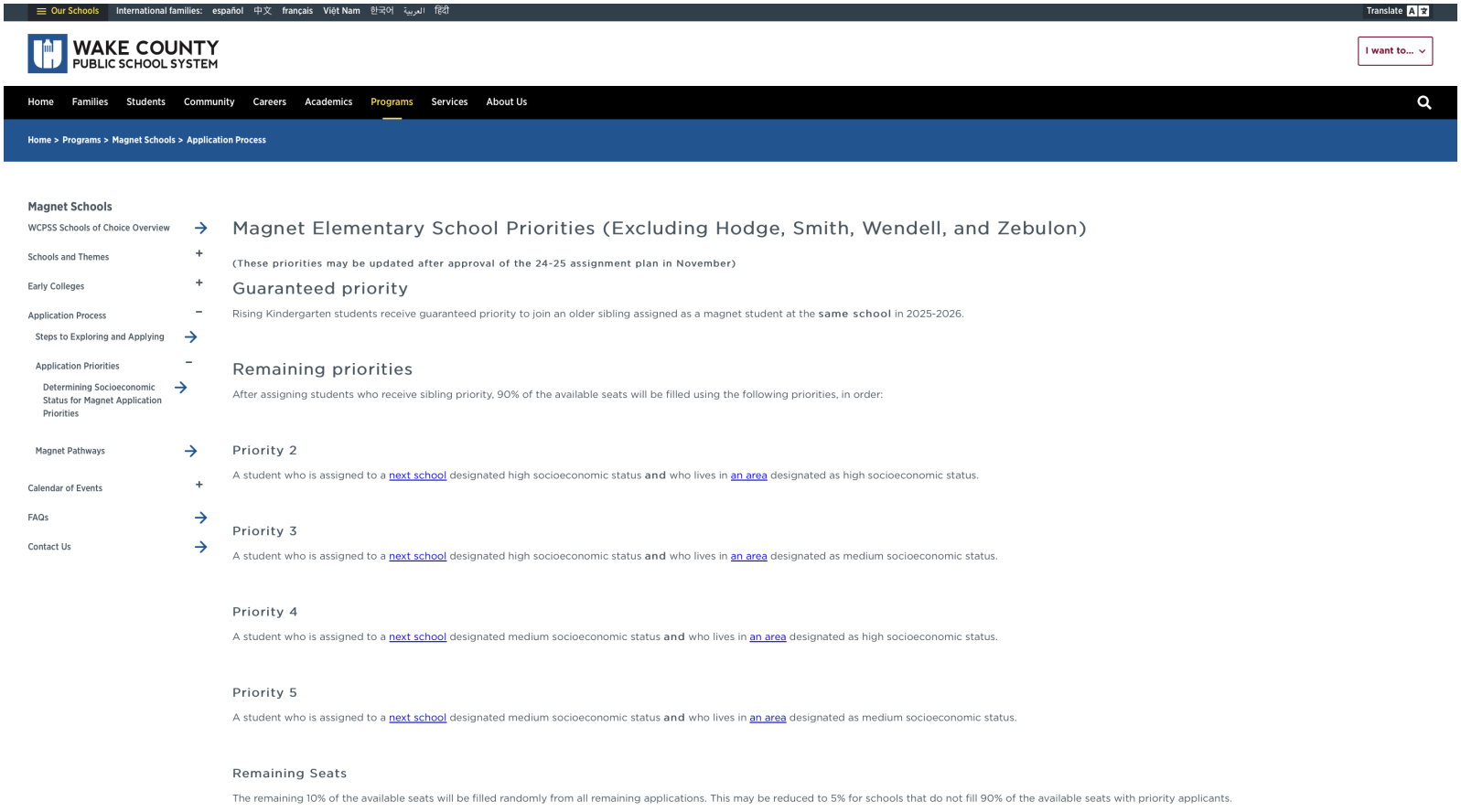
Introduction



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Magnet Schools

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Magnet Elementary School Priorities (Excluding Hodge, Smith, Wendell, and Zebulon)

(These priorities may be updated after approval of the 24-25 assignment plan in November)

Guaranteed priority

Rising Kindergarten students receive guaranteed priority to join an older sibling assigned as a magnet student at the **same school** in 2025-2026.

Remaining priorities

After assigning students who receive sibling priority, 90% of the available seats will be filled using the following priorities, in order:

Priority 2

A student who is assigned to a next school designated high socioeconomic status **and** who lives in an area designated as high socioeconomic status.

Priority 3

A student who is assigned to a next school designated high socioeconomic status **and** who lives in an area designated as medium socioeconomic status.

Priority 4

A student who is assigned to a next school designated medium socioeconomic status **and** who lives in an area designated as high socioeconomic status.

Priority 5

A student who is assigned to a next school designated medium socioeconomic status **and** who lives in an area designated as medium socioeconomic status.

Remaining Seats

The remaining 10% of the available seats will be filled randomly from all remaining applications. This may be reduced to 5% for schools that do not fill 90% of the available seats with priority applicants.

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One Seat, Respect Priorities

s

$c_s = 1$

$\succ_s$
1
2
3

One Seat, Respect Priorities

s

$c_s = 1$

$\succ_s$
1
2
3

Two Seats, Second Seat "Equal Opportunity"

s

$c_s = 2$

$\succ_s$
1
2
3

Two Seats, Second Seat "Equal Opportunity"

$$s$$

$$c_s = 2$$

$\succ_s$	T_s
1	1
2	3
3	2

Two Seats, Second Seat "Equal Opportunity"

$$s$$

$$c_s = 2$$

$\succ_s$	T_s
1	1
2	**3**
3	2

Two Seats, Second Seat "Equal Opportunity"

$$s$$

$$c_s = 2$$

$\succ_s$	T_s
1	1
2	**2**
3	3

Key Idea: Random Tie-Breaking

Define a tie-breaker T_s, use it for the Equal Opportunity seat.

Randomize over all possible tie-breakers.

This gives a sort of "procedural" fairness.

Similar approach to coarse priorities.

Ex Ante Approach

Mixtures of mechanisms satisfying ex post properties may violate ex ante notions.

Kesten and Ünver (2015)

- Deferred Acceptance with random tie-breaking is not *ex ante stable*.
- Propose the Fractional DA, which is *ex ante stable*.

Bogomolnaia and Moulin (2001)

- Random Serial Dictatorship is not *ex ante efficient*.
- Propose the Probabilistic Serial Rule, which is *sd efficient*.

Ex Ante Approach

Contribution

- Ex ante analysis of the problem with this requirement.
- Define ex ante notions of
  - *equal opportunity*
  - *adjusted no justified envy*
- Define new probabilistic mechanisms generalizing
  - DA
  - TTC
  - EADA

  to accommodate equal opportunity.
- Inspect their properties.

Ex Ante Approach

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- Ex ante analysis of the problem with this requirement.
- Define ex ante notions of
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  - *adjusted no justified envy*
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  - DA – FDAEO
  - TTC – TTCEO
  - EADA – EAFDAEO

  to accommodate equal opportunity.
- Inspect their properties.

Ex Ante Approach: Critique

"We do not reveal ex ante probabilities of assignments.
So students cannot check ex ante priority violation."

Response

- Current status quo mechanism and its information structure can be changed.
- Some implementations *post* the DA from random-tiebreaking for transparency, then find improvements.
- At a minimum, define and check ex ante fairness properties.

Literature

Discrete School Choice
Gale and Shapley [1962], Abdulkadiroğlu and Sönmez [2003], Ehlers and Klaus [2014, 2016], Kesten [2010], Tang and Yu [2014], Dur et al. [2018]

Probabilistic Assignment in School Choice
Roth et al. [1993], Kesten and Ünver [2015], Kojima and Manea [2010], Budish et al. [2013], Altuntaş and Phan [2022], Schlegel [2018]

Equal Opportunity
Basteck et al. [2021], Morrill and Hsu [2024]

Model

I	students
S	schools
P_i	student i's strict ranking over schools
c_s	school s capacity
$\succ_s$	school s strict priority order over students
e_s	each student's equal opportunity right at s where $\lvert I \rvert e_s \leq c_s$
$(I, S, P, c, \succ, e)$	a school choice problem with equal opportunity
$\triangle S$	the set of lotteries over S
x_i	an assignment for i where $x_i = (x_{is}) \in \triangle S$
x	an allocation $(x_i)_{i \in N}$, feasible if for each s, $\sum_{i \in N} x_{is} \leq c_s$
R_i^{sd}	sd-extension of R_i to preferences over $\triangle S$
φ	mechanism maps each problem to an allocation

Axioms: Equal Opportunity

Intuition
If student i wants to go only to school s,
then they should go there with *at least* e_s probability.

Student i should prefer their assignment to a lottery composed of

- going to s with probability e_s
- going to $\emptyset$ with probability $1 - e_s$

Axioms: Equal Opportunity

Intuition

- Student i has equal opportunity rights at *all* schools.
- $(e_{s_1}, e_{s_2}, ..., e_{s_n})$ does not generally constitute a lottery.
- Consider any lottery (x_{is}) where $x_{is} \leq e_s$, for each s.
- Student i has the equal opportunity "right" to each such lottery.

Axioms: Equal Opportunity

Equal Opportunity For each problem P, each student $i \in I$, and each $x_i \in EO_i = \{x_i \in \triangle S : \ x_{is} \leq e_s, \ \forall s \in S\}$,

$$\varphi_i(P) \ R_i \ x_i.$$

Axioms: Ex Ante No Justified Envy

Intuition

$$\begin{array}{c} R_i \\ \hline a \\ b \\ \vdots \end{array}$$

Axioms: Ex Ante No Justified Envy

Intuition

	R_i
a	0.5
b	0.5
	$\vdots$

Ex Ante No Justified Envy

Intuition

R_i	
a	0.5
b	0.5
$\vdots$	

$\succ_a$
i
j
$\vdots$

Axioms: Ex Ante No Justified Envy

Intuition

If student i would prefer to increase their probability of going to school s, then student j with lower priority should not receive positive probability of going to s.

Axioms: Ex Ante No Justified Envy

Kesten and Ünver [2015]:

An allocation x causes **justified envy** at P if there is $i, j \in I$ and $s, t \in S$ such that

- $i \succ_s j$,
- $s \, P_i \, t$,
- $x_{js} > 0$, and
- $x_{it} > 0$.

A mechanism satisfies **ex ante no justified envy** if there is no justified envy at any problem.

Axioms: Ex Ante No Justified Envy

Incompatibility with EO

R_i	R_j
a	a
b	$\vdots$
$\vdots$	

$\succ_a$
i
j
$\vdots$

Axioms: Ex Ante No Justified Envy

Incompatibility with EO

R_i	R_j
a	a $.1$
b	$\vdots$
$\vdots$	

$\succ_a$
i
j
$\vdots$

Axioms: Ex Ante No Justified Envy

Incompatibility with EO

R_i		R_j	
a	.9	a	.1
b		$\vdots$	
$\vdots$			

$\succ_a$
i
j
$\vdots$

Axioms: Ex Ante No Justified Envy

Kesten and Ünver [2015]:

An allocation x causes **justified envy** at P if there is $i, j \in I$ and $s, t \in S$ such that

- $i \succ_s j$,
- $s \, P_i \, t$,
- $x_{js} > 0$, and
- $x_{it} > 0$.

A mechanism satisfies **ex ante no justified envy** if there is no justified envy at any problem.

Axioms: Ex Ante EO Adjusted No Justified Envy

This paper:

An allocation x causes **EO adjusted justified envy** at P if there is $i, j \in I$ and $s, t \in S$ such that

- $i \succ_s j$,
- $s \, P_i \, t$,
- $x_{js} > e_s$, and
- $x_{it} > 0$.

A mechanism satisfies **ex ante EO adjusted no justified envy** if there is no EO adjusted justified envy at any problem.

Mechanisms

- DA
- EADA
- TTC

Mechanisms

- DA - FDA
- EADA
- TTC

Mechanisms

- DA - FDAEO
- EADA
- TTC

Mechanisms

- DA - FDAEO
- EADA - EAFDAEO
- TTC - TTCEO

Mechanisms: FDAEO

Step 1.

- Each student demands one unit of their most preferred school.
- Each school specifies specifies a tentative probability of acceptance:
  1. Each student receives e_s (or their demand if lower).
  2. Distribute remaining capacity c_s according to priority $\succ_s$.
- Call $x_{is}(1)$ what i receives at s at Step 1.

Step $k \geq 2$.

- Each student i additionally demands $1 - \sum x_{is}$ of their most preferred school that has not rejected their demand.
- Each school specifies a tentative probability of acceptance:
  1. Each student receives e_s (or their demand if lower).
  2. Distribute remaining capacity c_s according to priority $\succ_s$.
- Call $x_{is}(k)$ what i receives at s at Step k.

Mechanisms: FDAEO

Theorem 1 The FDAEO mechanism terminates in finitely many steps, and satisfies *equal opportunity* and *EO adjusted no justified envy*.

Remark The FDAEO is

- not (sd) efficient,
- not *constrained efficient* within the EO and EO adjusted no justified envy set, and
- not (sd) *strategyproof*.

Mechanisms: EAFDAEO

Intuition

- Round 1. Run FDAEO.
- Round $k \geq 2$.
  - Identify *underdemanded* schools [Tang and Yu, 2014].
  - Fix assignment of these schools.
  - Remove these schools from agents' preference ranking.
  - For this reduced problem, run FDAEO.

Mechanisms: EAFDAEO

Consider a problem P and an allocation x.

School s is **underdemanded** at x if no student $i \in I$ prefers s to any school with which they have positive probability at x_i.

Intuitively, the set of underdemanded schools in $FDAEO(P)$ proposed to at the last step of the FDAEO algorithm.

Mechanisms: EAFDAEO

Round 1
Run the FDAEO.

Round $k \geq 2$.

- Identify *underdemanded* schools [Tang and Yu, 2014].
- Fix assignment of these schools.
- Remove these schools from agents' preference ranking.
- Run FDAEO for this reduced problem.

Mechanisms: EAFDAEO

Lemma At each round k, the k-FDAEO allocation weakly Pareto-dominates the $k-1$-FDAEO allocation.

Remark This may violate some students priorities.

Theorem 2 The EAFDAEO mechanism satisfies EO and *sd efficiency*.

Mechanisms: EAFDAEO and Consent

Kesten [2010]:

Intuition
If student i consents, and $i \succ_s j$, then j can receive s possibly violating i's priority.

Mechanisms: EAFDAEO and Consent

This paper:

Intuition
If student i does not consent, and $i \succ_s j$, then j can demand only up to e_s.

Mechanisms: EAFDAEO and Consent

A school s is **constrained underdemanded** if whenever there is student i and school t with $x_{it} > 0$ and $s\, P_i\, t$, then there is non-consenting student j such that $j \succ_s i$, and for each school s' such that $s\, P_j\, s'$, $x_{js'} = 0$.

R_i	R_j
s	$\vdots$
$t > 0$	s
$\vdots$	$\vdots$

$\succ_s$
j
i
$\vdots$

Remark Constrained underdemanded are the schools proposed to at the last step of the FDAEO each round.

Mechanisms: EAFDAEO

Round 1
Run the FDAEO.

Round $k \geq 2$.

- Identify constrained underdemanded schools.
- Fix assignment of these schools.
- Remove these schools from agents' preference ranking.
- If student i does not consent, then for each school s (that is not the last-ranked school that they receive positive probability of), and student j with lower priority at $\succ_s$, limit j's demand of s to e_s.
- Run FDAEO for this reduced problem.

Mechanisms: EAFDAEO

Lemma The EAFDAEO mechanism respects non-consenting students' priorities.

Theorem 3 The EAFDAEO is efficient within the set of allocations satisfying EO and EO adjusted no justified envy for non-consenting students.

Mechanisms: TTCEO

Intuition

- If student i has top priority at a school, then they can *trade* their seat with another student.
- Now, each student has a *probabilistic* share of s that they can trade.
- Each priority order is also composed of two parts:
  - all students appear with trading right e_s
  - all students appear in order $\succ_s$ with "remaining" trading right

Mechanisms: TTCEO

$\hat{\succ}_s$
1
2
$\vdots$
n
$\succ_s (1)$
$\vdots$
$\succ_s (n)$

Mechanisms: TTCEO

Step 1

- Each student points at their most preferred school with weight 1.
- Each school s points to their highest priority agent in $\hat{\succ}_s$ with weight e_s.
- Trade minimum probabilities along cycles.

Step $k \geq 2$

- Each student points at their most preferred school with remaining demand.
- Each school s points to their highest priority agent in $\hat{\succ}_s$. Weight is remaining e_s, or remaining capacity of s if in second part of $\hat{\succ}_s$.
- Trade minimum probabilities along cycles.

Mechanisms: TTCEO

Theorem 4 TTCEO satisfies EO, and sd efficiency.

Remark TTCEO violates EO adjusted no justified envy and strategy-proofness.

Impossibility

Remark

- There is no EO, efficient, and strategy-proof mechanism.
- There is no EO NJE efficient and strategy-proof mechanism.

Summary

	DAEO	EDAEO			TTCEO
		full	*partial*	*no*	
EO	+	+	+	+	+
EO NJE	+	−	−	+	−
Eff	−	+	−	−	+
EO NJE Eff	−	−	−	+	−
EO NJE Eff wrt NC	−	+	+	+	−
STP	−	−	−	−	−

Summary

	DA1C	DAC	IA1C	MEO‡	RP$^{\diamond}$
EO	$-$	$+$	$-$	$+$	$+$
EO NJE	$-$	$-$	$-$	$+$	$-$
Eff	$-$	$-$	$+$	$-$	$-$
EO NJE Eff	$-$	$-$	$-$	$-$	$-$
EO NJE Eff wrt NC	$-$	$-$	$-$	$-$	$-$
STP	$-$	$+$	$-$	$+$	$+$

Conclusion

Thank you!

Contribution

- Ex ante analysis of equal opportunity in school choice problem.
- Probabilistic generalizations of central mechanisms in the literature.

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