

School Finance, Peers, and Choice

William Phan¹ Ryan Tierney² Yu Zhou³

¹North Carolina State University

²University of Southern Denmark

³Nagoya University

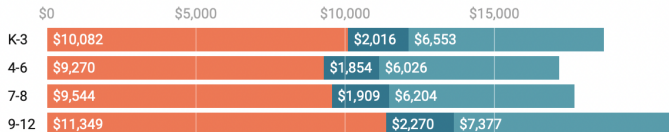
Mechanism Design Without Transfers
Hausdorff Research Institute for Mathematics
University of Bonn

California Education Funding Formula

LCFF Per-Student Rates

2022-23 Enacted Budget (revised August 2022)

Base Supplemental Concentration



Transitional Kindergarten 2022-23 add-on rate per student in attendance (ADA): \$2,813

Chart: Ed100 Lesson 8.5 • Source: [California Department of Education](#) • [Get the data](#) • [Embed](#) • Created with [Datawrapper](#)

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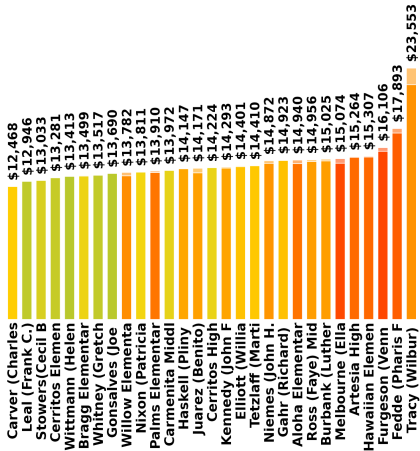
in

🐦

Funding

- ▶ California: \$7.5 billion (10% of LCFF)
- ▶ North Carolina: \$1.6 billion (7%)
- ▶ Texas: \$3.4 billion (6%)

Funding



California ABC Unified School District 2020-2021

Funding

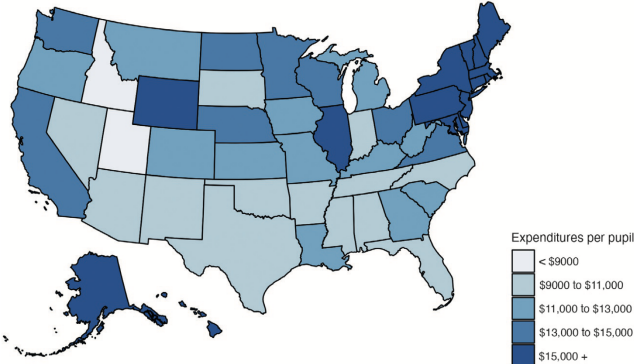


FIG. 3

Per-pupil expenditure by state, 2019.

Source: NCES 2021 digest (https://nces.ed.gov/programs/digest/2021menu_tables.asp), table 236.65.

Funding

Table 1 Distribution of funding source makeup with representative states, 2019 (percent)

Funding source	Mean	Minimum	Maximum
Local	42.26	2.10 (Hawaii)	91.97 (DC)
State	50.07	26.57 (Illinois)	90.29 (Vermont)
Federal	8.63	4.12 (New Jersey)	15.44 (Alaska)

Source: NCES 2021 digest (https://nces.ed.gov/programs/digest/2021menu_tables.asp), table 235.20.

Literature

School Finance

\$ vs. Outcomes

Sorting & Peer Effects

Neighbors, Peers vs. Preferences or Outcomes

School Choice

Designing assignment mechanisms

Literature

School Finance

Handel and Hanushek [2023], Fiszbein and Schady [2009], Angrist, Bettinger, Bloom, King, and Kremer [2002], Benhassine, Devoto, Duflo, Dupas, and Pouliquen [2015], Neilson [2021], Bobba, Ederer, Leon-Ciliotta, Neilson, and Nieddu [2021]

Sorting & Peer Effects

Epple and Romano [1998], Hoxby [2001, 2003], Rothstein [2006], Abdulkadiroğlu, Angrist, and Pathak [2014], Abdulkadiroğlu, Pathak, Schellenberg, and Walters [2020], Avery and Pathak [2021]

School Choice

Abdulkadiroğlu and Sönmez [2003], Abdulkadiroğlu, Pathak, and Roth [2009], Leshno [2022]

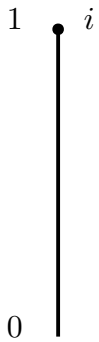
Contribution

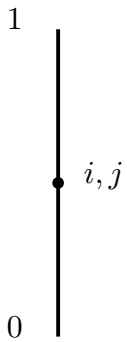
1. A *resource* model of school choice.
2. Propose *enrollment-based funding equilibria*.
3. Applications
 - 3.1 Funding in schools
 - 3.2 Peer effects
 - 3.3 Vouchers
4. New proof techniques
 - 4.1 crowding and funding drive the difficulties
 - 4.2 not Tarski fixed point
 - 4.3 not Decomposition Lemma

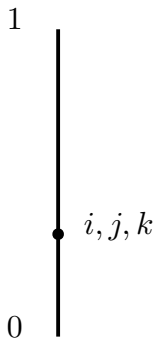
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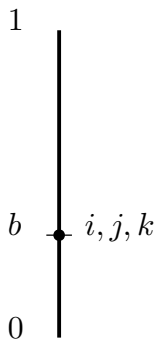


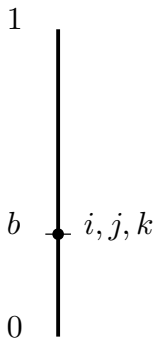
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$$\frac{1}{b} = \textit{capacity}$$

1



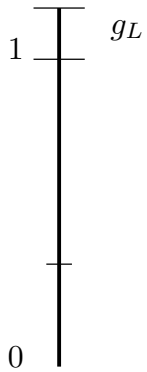
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i

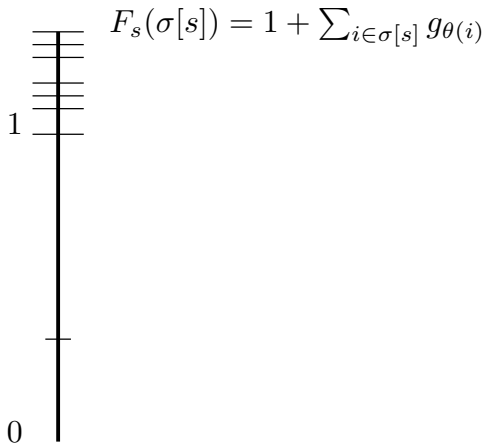


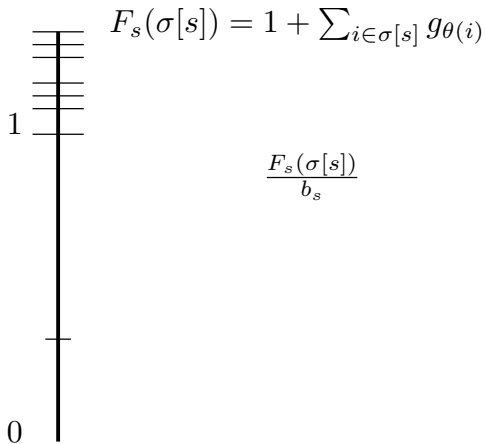
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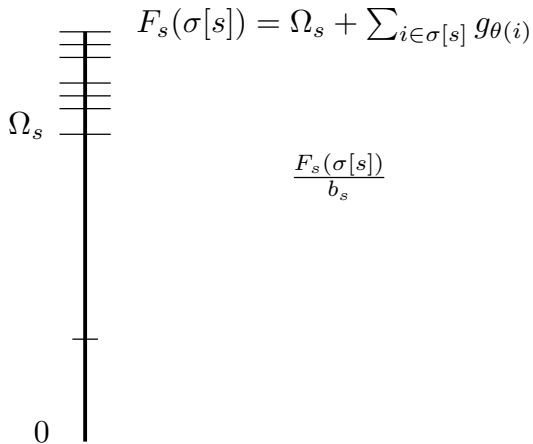
California: general staff hiring, general teacher raises.

North Carolina: general student resources.

Texas: half of the Compensatory Allotment can be used for general spending.







s_1



s_2



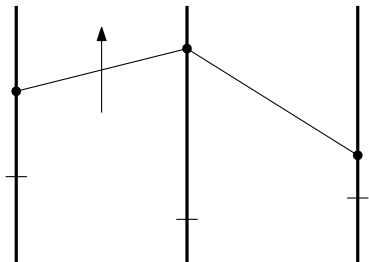
s_3



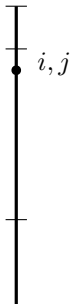
s_1

s_2

s_3



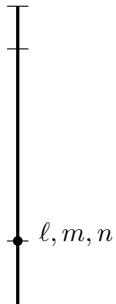
s_1



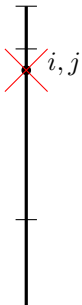
s_2



s_3



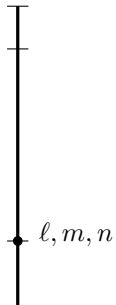
s_1



s_2



s_3



s_1



i, j

s_2



k

s_3



l, m, n

s_1



i, j, k

s_2



s_3



l, m, n

A **student's externality** has 3 parts:

1. negative crowding effect
2. positive funding
3. negative/positive personal peer effect

Model

L, H	low and high grant students
S	schools
$\mathbb{R}_+ \times S$	consumption space of students
R_i	preference relation of student i
Ω_s	initial capital of school s
b_s	minimum distribution of school s
g_L	low grant
g_H	high grant
\succ_s	priority order of school s over students

$(L, H, S, \Omega, b, g, \succ, R)$

school choice with funding problem

ρ_s	distribution of school s
$\sigma(i)$	match of student i
(ρ, σ)	allocation

Model

Total Funding

$$F_s(\sigma) = \Omega_s + \sum_{i \in \sigma[s]} g_{\theta(i)}$$

An **allocation** (ρ, σ) is such that for each school s ,

1. (Distribution Feasibility)

$$\rho_s \cdot |\sigma[s]| \leq F_s(\sigma).$$

2. (Minimum Bound)

$$\rho_s \geq b_s.$$

Axiom

Fairness

No Envy at Interior + No Justified Envy at Boundary

Axiom

Fairness

If student i prefers school s to their match at the current level, then s is

1. full, and
2. all students at s have higher priority.

Axiom

Fairness

If student i prefers school s to their match at the current level, then s is

1. is at minimum b_s , and
2. all students at s have higher priority.

Axiom

Fairness:

$$(\rho, s) P_i (\rho, \sigma(i)) \Rightarrow \begin{array}{l} 1. \rho_s = b_s, \text{ and} \\ 2. \text{ for each } j \in \sigma[s], j \succ_s i, \end{array}$$

Axiom

Fairness:

$$(\rho, s) P_i (\rho, \sigma(i)) \Rightarrow \begin{array}{l} 1. \rho_s = b_s, \text{ and} \\ 2 \text{ for each } j \in \sigma[s], \\ \quad \text{a. } j \succ_s i, \text{ or} \\ \quad \text{b. } (\sigma[s] \setminus \{i\}) \cup \{j\} \text{ is not feasible.} \end{array}$$

Enrollment-Based Funding Equilibrium (EBFE)

An EBFE is an allocation (ρ, σ) such that:

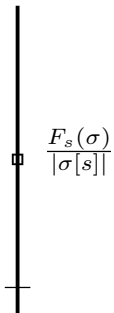
1. (IR + Fairness) (ρ, σ) is *individually rational* and *fair*.
2. (Exhaustive) For each school s with $\sigma[s] \neq \emptyset$,

$$\left\lfloor \frac{F_s(\sigma)}{\rho_s} \right\rfloor = |\sigma[s]|.$$

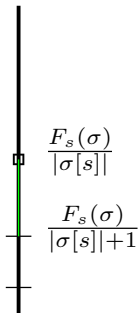
3. (Inferior Empty Schools) For each school s with $\sigma[s] = \emptyset$, $\rho_s = \Omega_s$, and for each $i \in N$,

$$(\rho, \sigma(i)) P_i (\rho, s).$$

Exhaustiveness Discrepancy


$$\frac{F_s(\sigma)}{|\sigma[s]|}$$

Exhaustiveness Discrepancy



EBFE Is a Walrasian Concept

An EBFE is an allocation (ρ, σ) such that:

1. (Fairness) Consumer maximization.
2. (Exhaustive) Market clearing with one seat error.
3. (Inferior Empty Schools) Set price of unassigned good to zero.

Price Equilibrium with Rigidities

Drèze [1975], Talman and Yang [2008], Andersson and Svensson [2014], Herings [2018].

Existence

A preference is **linear** if $(r, s) R_i (g, t)$ implies that there is valuation function $v_i : S \rightarrow \mathbb{R}_+$, such that

$$(r, s) R_i (g, t) \iff rv_i(s) \geq gv_i(t).$$

Main Results: Existence & Structural Properties

Theorem 1 On the NCBI linear domain, EBFE exist.

Proposition 1 The set of *fair* allocations forms an upper lattice.

Theorem 2 Given a profile from the NCBI linear domain, there is a student-optimal EBFE.

Main Results: Existence & Structural Properties

Proof Intuition:

- ▶ *Fair* allocations trivially exist.
- ▶ Prove upper lattice of *fair* set, so there is a student-optimal *fair* allocation.
- ▶ This allocation may fail exhaustiveness, so we must augment the matching to find an EBFE.

Main Results: Some Things Break

- ▶ Set of distributions that are part of some EBFE do not form a lattice.
- ▶ Exists *fair* that are not EBFE.
- ▶ No Rural Hospital / Lone Wolf theorems.
- ▶ Manipulability of student-optimal EBFE mechanism.

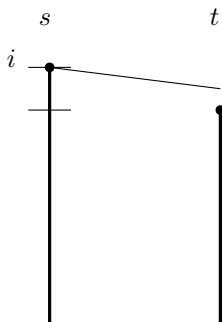
No Rural Hospital Theorem

s

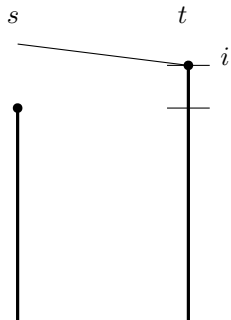
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No Rural Hospital Theorem



No Rural Hospital Theorem



Impossibility

Proposition 3 A mechanism that is a selection from the student-optimal EBFE correspondence is not *strategy-proof*.

General Model

L, H	low and high grant students
S	schools
$\mathbb{R}_+ \times S$	consumption space of students
R_i	preference relation of student i
Ω_s	initial capital of school s
b_s	minimum distribution of school s
g_L	low grant
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\succ_s	priority order of school s over students

$(L, H, S, \Omega, b, g, \succ, R)$ **school choice with funding problem**

General Model

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$(I, S, Ch, \Omega, b, g, v, R)$

school choice with funding problem

General Model

L, H

S

$\mathbb{R}_+ \times S$

R_i

Ω_s

b_s

g_L

g_H

\succ_s

low and high grant students

schools

consumption space of students

preference relation of student i

initial capital of school s

minimum distribution of school s

low grant

high grant

priority order of school s over students

$(I, S, Ch, \Omega, b, g, v, R)$

school choice with funding problem

General Model

I students

S schools

$\mathbb{R}_+ \times S$ consumption space of students

R_i preference relation of student i

Ω_s initial capital of school s

b_s minimum distribution of school s

g_i total externality of i

\succsim_s priority order of school s over students

$(I, S, Ch, \Omega, b, g, v, R)$ **school choice with funding problem**

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Ω_s	initial capital of school s
b_s	minimum distribution of school s
g_i	total externality of i
v_i	voucher of i
Ch_s	preferences of school s over students

$(I, S, Ch, \Omega, b, g, v, R)$

school choice with funding problem

School Choice Functions

The choice function $Ch_s(A) \subseteq A$ of each school s satisfies:

- ▶ Substitutability

$$B \supseteq A \implies Re_s(B) \supseteq Re_s(A)$$

- ▶ Independence of Rejected Students

$$A' \subseteq Re_s(A) \implies Ch_s(A \setminus A') = Ch_s(A)$$

- ▶ Budget Constraint

$$b_s |Ch_s(A)| \leq F_s(Ch_s(A))$$

Fair Lattice

Proposition 1 (Fair Upper Lattice) Consider two *fair* allocations (ρ, σ) and (γ, τ) for a preference profile from the NCBI linear domain. Let $\zeta = \rho \vee \gamma$. There is a matching μ such that (ζ, μ) is a *fair* allocation, and moreover, for each $i \in N$,

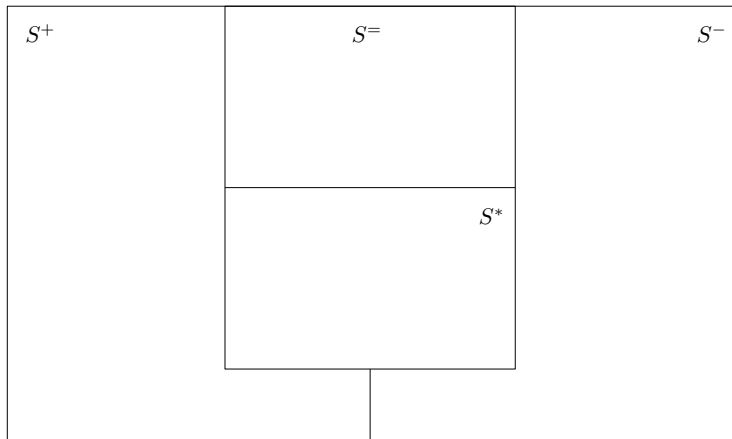
$$(\zeta, \mu(i)) R_i \max_{R_i} \{(\rho, \sigma(i)), (\gamma, \tau(i))\}.$$

Fair Lattice

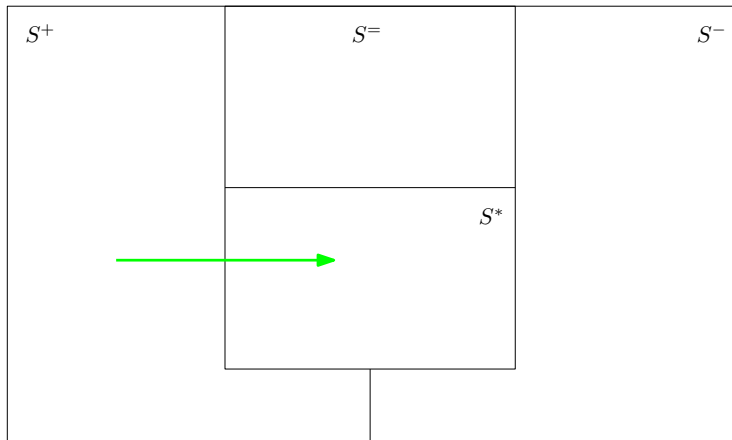
Sketch Proof:

1. Pick two fair allocations.
2. Define Transfer graph.
3. Implement weakly welfare-increasing cycles.
4. (Key) Make sure you maintain feasibility.

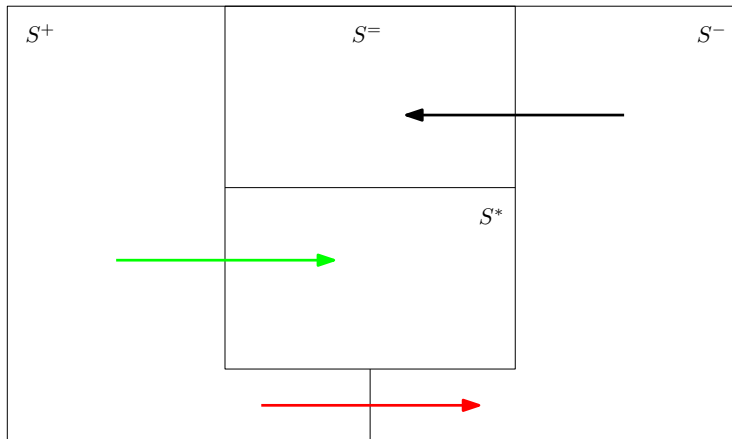
Fair Lattice



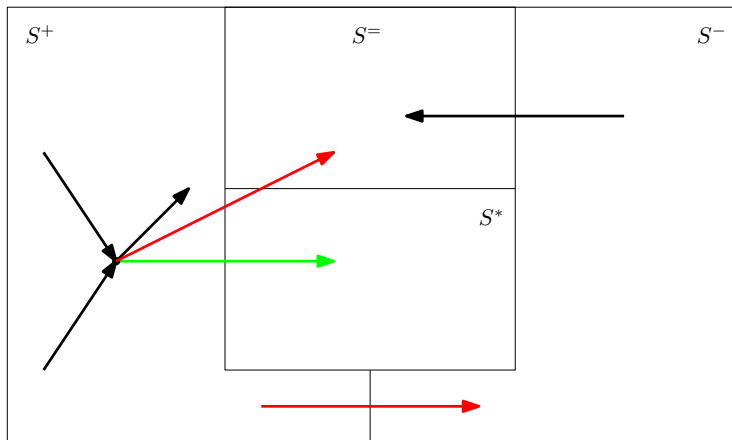
Fair Lattice



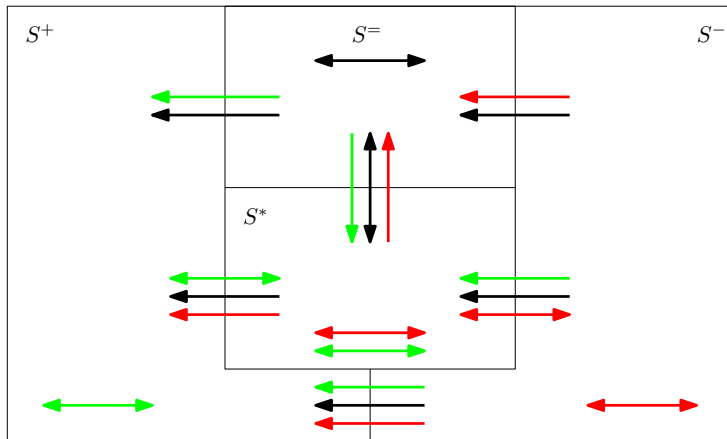
Fair Lattice



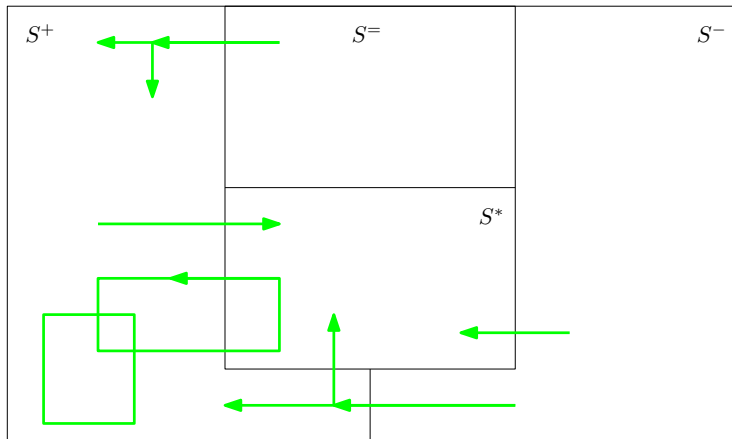
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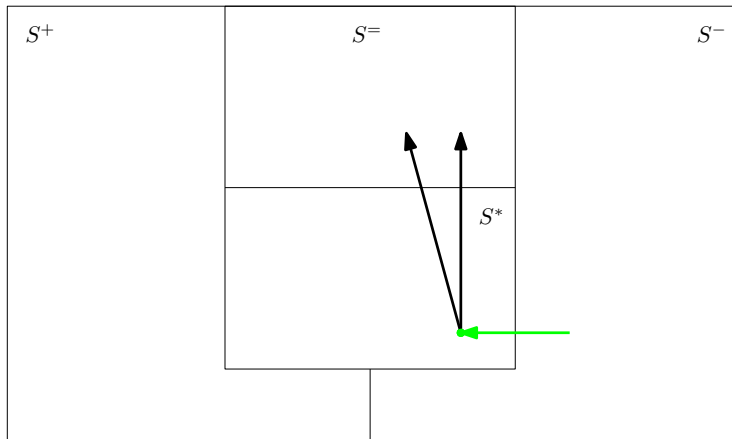
Fair Lattice



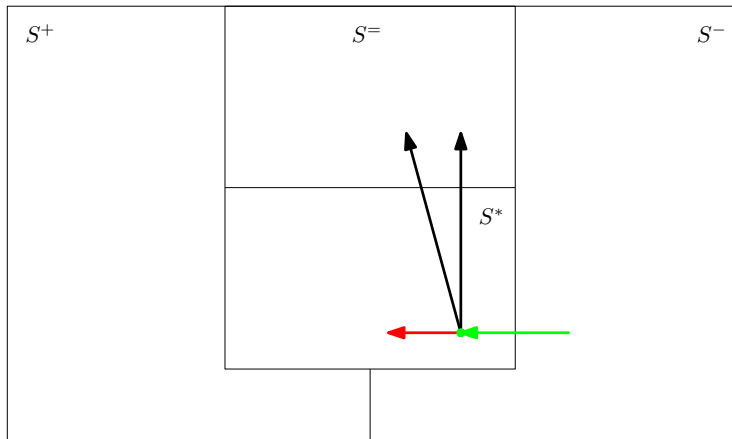
Fair Lattice



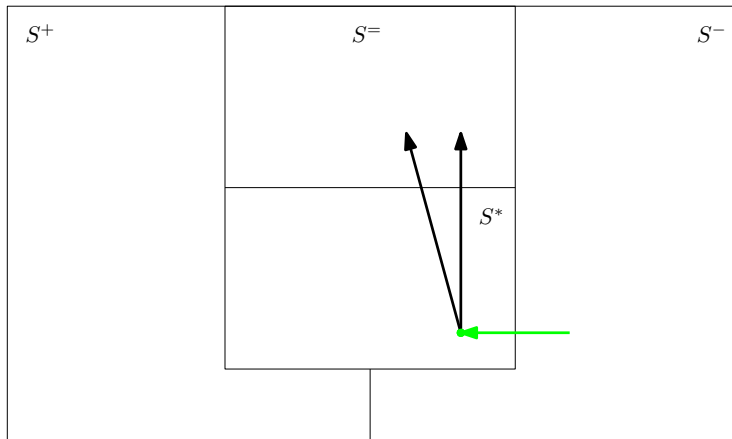
Fair Lattice



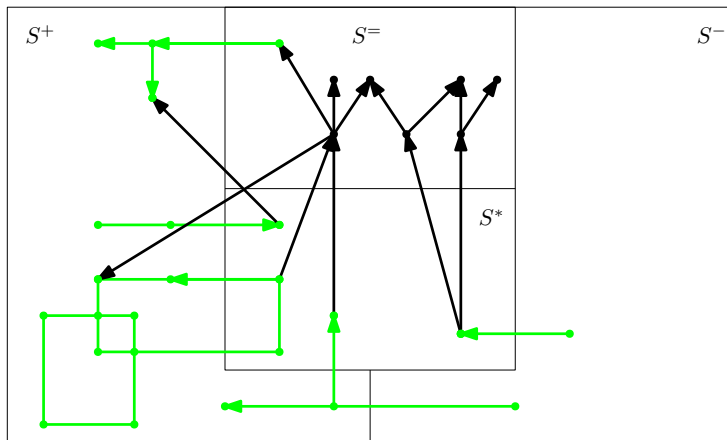
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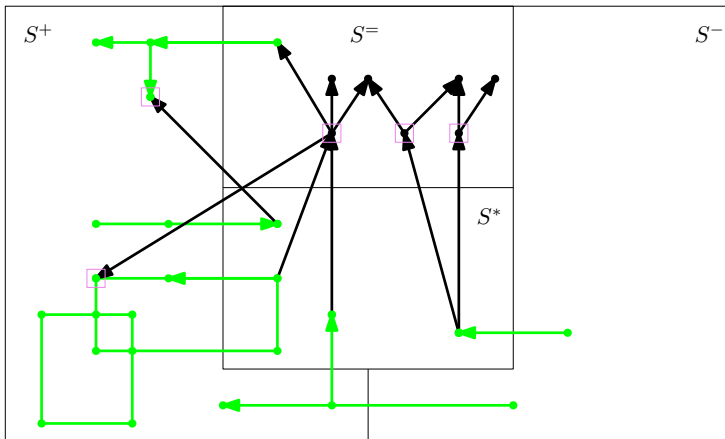
Fair Lattice



Fair Lattice



Fair Lattice



Conclusion

Thank you!

School Finance

Sorting & Peer Effects

School Choice

School Choice

Abdulkadiroğlu and Sönmez [2003], Phan et al. [2024].

Matching with Externalities

Sasaki and Toda [1996], Hafalir [2008], Bando [2012], Bando [2014], Dutta and Massó [1997], Echenique and Yenmez [2007], Roth [1984], Aldershof and Carducci [1996], Roth and Peranson [1999], Klaus and Klijn [2005], Kojima et al. [2013], Ashlagi et al. [2014], Dur and Wiseman [2019], Dur et al. [2022], Pycia [2012], Rostek and Yoder [2020], Pycia and Yenmez [2023].

Literature

Price Equilibrium with Rigidities

Drèze [1975], Talman and Yang [2008], Andersson and Svensson [2014], Herings [2018].

Price As Cutoff Vector

Balinski and Sönmez [1999], Sönmez and Ünver [2010], Azevedo and Leshno [2016], Dur and Morrill [2018].

Crowding

Tierney [2019].

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