

School Finance, Peers, and Choice

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A Critique

“The matching literature is driven by the urge to prove theorems about formal structures rather than any empirical phenomenon. School choice does not increase welfare in any way.”

A Critique

school **choice**

A Critique

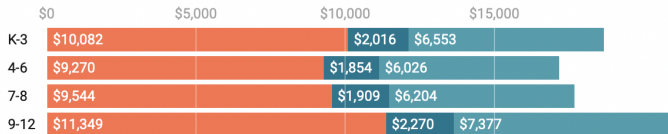
school choice

California Education Funding Formula

LCFF Per-Student Rates

2022-23 Enacted Budget (revised August 2022)

Base Supplemental Concentration



Transitional Kindergarten 2022-23 add-on rate per student in attendance (ADA): \$2,813

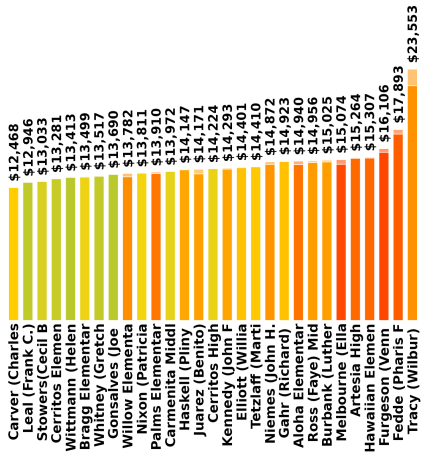
Chart: Ed100 Lesson 8.5 • Source: [California Department of Education](#) • [Get the data](#) • [Embed](#) • Created with [Datawrapper](#)



Funding

- ▶ California: \$7.5 billion (10% of LCFF)
- ▶ North Carolina: \$1.6 billion (7%)
- ▶ Texas: \$3.4 billion (6%)

Funding



California ABC Unified School District 2020-2021

Funding

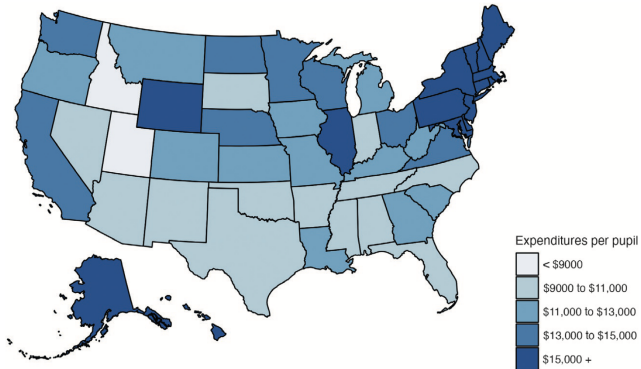


FIG. 3

Per-pupil expenditure by state, 2019.

Source: NCES 2021 digest (https://nces.ed.gov/programs/digest/2021menu_tables.asp), table 236.65.

Funding

Table 1 Distribution of funding source makeup with representative states, 2019 (percent)

Funding source	Mean	Minimum	Maximum
Local	42.26	2.10 (Hawaii)	91.97 (DC)
State	50.07	26.57 (Illinois)	90.29 (Vermont)
Federal	8.63	4.12 (New Jersey)	15.44 (Alaska)

Source: NCES 2021 digest (https://nces.ed.gov/programs/digest/2021menu_tables.asp), table 235.20.

Literature

School Finance

\$ vs. Outcomes

Sorting & Peer Effects

Neighbors, Peers vs. Preferences or Outcomes

School Choice

Designing assignment mechanisms

Literature

School Finance

Krueger [1999], Chetty, Friedman, Hilger, Saez, Schanzenbach, and Yagan [2011], Jackson, Johnson, and Persico [2016], Lafortune, Rothstein, and Schanzenbach [2018], Angrist, Bettinger, Bloom, King, and Kremer [2002], Neilson [2021], Bobba, Ederer, Leon-Ciliotta, Neilson, and Nieddu [2021]

Sorting & Peer Effects

Epple and Romano [1998], Hoxby [2001, 2003], Rothstein [2006], Abdulkadiroğlu, Angrist, and Pathak [2014], Abdulkadiroğlu, Pathak, Schellenberg, and Walters [2020], Avery and Pathak [2021]

School Choice

Gale and Shapley [1962], Abdulkadiroğlu and Sönmez [2003], Abdulkadiroğlu, Pathak, and Roth [2009], Leshno [2022]

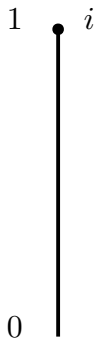
Contribution

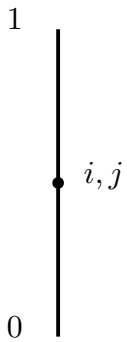
1. A *resource* model of school choice.
2. Propose *enrollment-based funding equilibria*.
3. Applications
 - 3.1 Funding in schools
 - 3.2 Peer effects
 - 3.3 Vouchers
4. New proof techniques
 - 4.1 crowding and funding drive the difficulties
 - 4.2 not Tarski fixed point
 - 4.3 not Decomposition Lemma

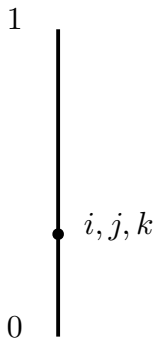
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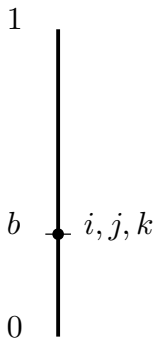
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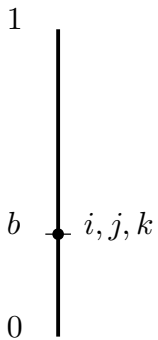












$$\frac{1}{b} = \textit{capacity}$$



1



0

i

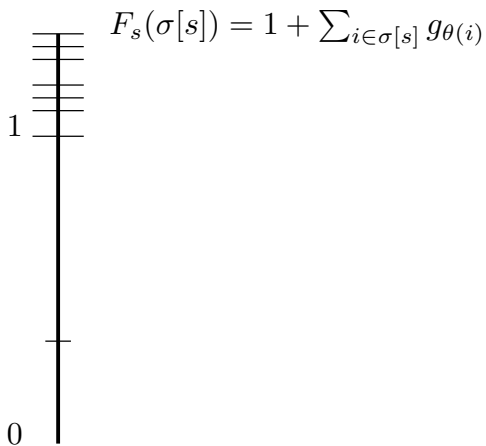


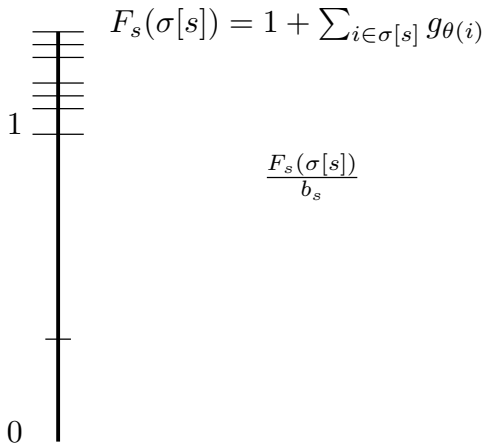
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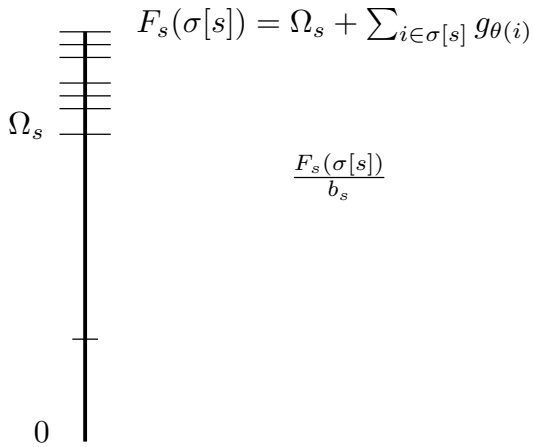
California: general staff hiring, general teacher raises.

North Carolina: general student resources.

Texas: half of the Compensatory Allotment can be used for general spending.







s_1



s_2



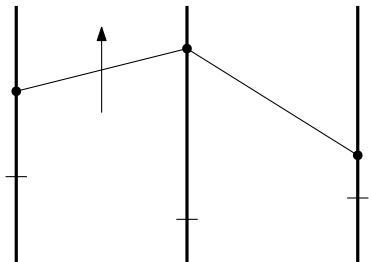
s_3



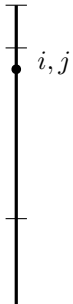
s_1

s_2

s_3



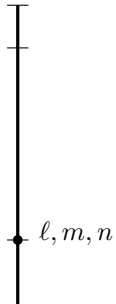
s_1



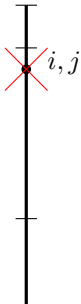
s_2



s_3



s_1



s_2



s_3



s_1



i, j

s_2



k

s_3



ℓ, m, n

s_1



i, j, k

s_2



s_3



ℓ, m, n

A **student's externality** has 3 parts:

1. negative crowding effect
2. positive funding
3. negative/positive personal funding

A **student's externality** has 3 parts:

1. negative crowding effect
2. positive funding
3. negative/positive personal peer effect

Model

I	students
S	schools
$\mathbb{R}_+ \times S \times \mathbb{R}_+$	consumption space of students
R_i	preference relation of student i
Ω_s	initial capital of school s
b_s	minimum distribution of school s
g_i	total externality of i
v_i	voucher of i
Ch_s	preferences of school s over students
$(I, S, Ch, \Omega, b, g, v, R)$	school choice with funding problem

Model

Total Funding

$$F_s(\sigma) = \Omega_s + \sum_{i \in \sigma[s]} g_i$$

An **allocation** (ρ, σ) is such that for each school s ,

1. (Distribution Feasibility)

$$\rho_s \cdot |\sigma[s]| \leq F_s(\sigma).$$

2. (Minimum Bound)

$$\rho_s \geq b_s.$$

School Choice Functions

The choice function $Ch_s(A) \subseteq A$ of each school s satisfies:

- ▶ Substitutability

$$B \supseteq A \implies Re_s(B) \supseteq Re_s(A)$$

- ▶ Independence of Rejected Students

$$A' \subseteq Re_s(A) \implies Ch_s(A \setminus A') = Ch_s(A)$$

- ▶ Budget Constraint

$$b_s |Ch_s(A)| \leq F_s(Ch_s(A))$$

Axiom

Fairness

No Envy at Interior + No Justified Envy at Boundary

Axiom

Fairness

If student i prefers school s to their match at the current level, then

1. s is full, and
2. would not replace some assigned student with i .

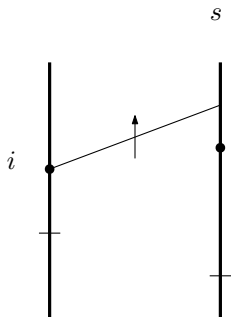
Axiom

Fairness

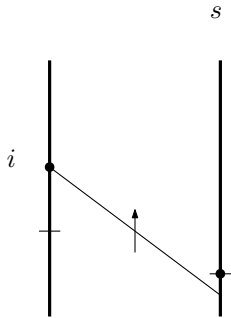
If $(\rho, s) P_i (\rho, \sigma(i))$, then

1. $\rho_s = b_s$, and
2. there is no $j \in \sigma[s]$ such that
 - 2.1 $i \in Ch(\sigma[s] \cup \{i\})$, and
 - 2.2 $j \notin Ch(\sigma[s] \cup \{i\})$.

Axiom



Axiom



Axiom

Fairness contains no *efficiency* content.

Explore the fair correspondence.

Idea (draw)

1. Start from a fair allocation.
2. Perturb upwards resource usage.
3. Connectedness Property.

Enrollment-Based Funding Equilibrium (EBFE)

An EBFE is an allocation (ρ, σ) such that:

1. (IR + Fairness) (ρ, σ) is *individually rational* and *fair*.
2. (Exhaustive) For each school s with $\sigma[s] \neq \emptyset$,

$$\left\lfloor \frac{F_s(\sigma)}{\rho_s} \right\rfloor = |\sigma[s]|.$$

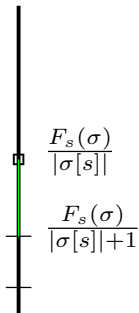
3. (Inferior Empty Schools) For each school s with $\sigma[s] = \emptyset$, $\rho_s = \Omega_s$, and for each $i \in N$,

$$(\rho, \sigma(i)) P_i (\rho, s).$$

Exhaustiveness Discrepancy


$$\frac{F_s(\sigma)}{|\sigma[s]|}$$

Exhaustiveness Discrepancy



EBFE Is a Walrasian Concept

An EBFE is an allocation (ρ, σ) such that:

1. (Fairness) Consumer maximization.
2. (Exhaustive) Market clearing with one seat error.
3. (Inferior Empty Schools) Set price of unassigned good to zero.

Price Equilibrium with Rigidities

Drèze [1975], Talman and Yang [2008], Andersson and Svensson [2014], Herings [2018].

Existence

A preference is **linear** if $(r, s) R_i (g, t)$ implies that there is valuation function $v_i : S \rightarrow \mathbb{R}_+$, such that

$$(r, s) R_i (g, t) \iff rv_i(s) \geq gv_i(t).$$

Main Results: Existence & Structural Properties

Theorem 1 On the NCBI linear domain, EBFE exist.

Proposition 1 On the NCBI linear domain, the set of *fair* allocations forms an upper lattice.

Theorem 2 Given a profile from the NCBI linear domain, there is a student-optimal EBFE.

Main Results: Existence & Structural Properties

Proof Intuition:

- ▶ *Fair* allocations trivially exist.
- ▶ Prove upper lattice of *fair* set, so there is a student-optimal *fair* allocation.
- ▶ This allocation may fail exhaustiveness, so we must augment the matching to find an EBFE.

Main Results: Some Things Break

- ▶ Set of distributions part of some EBFE do not form a lattice.
- ▶ Exists *fair* that are not EBFE.
- ▶ No Rural Hospital / Lone Wolf theorems.
- ▶ Manipulability of student-optimal EBFE mechanism.

Fair Lattice

Proposition 1 (Fair Upper Lattice) Consider two *fair* allocations (ρ, σ) and (γ, τ) for a preference profile from the NCBI linear domain. Let $\zeta = \rho \vee \gamma$. There is a matching μ such that (ζ, μ) is a *fair* allocation, and moreover, for each $i \in N$,

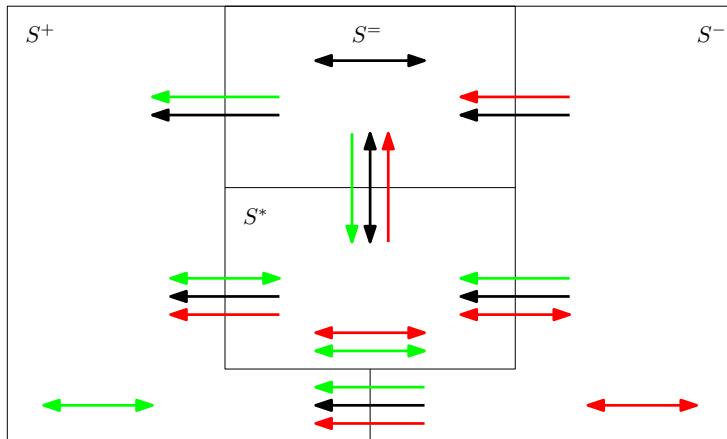
$$(\zeta, \mu(i)) \succeq_{R_i} \max\{(\rho, \sigma(i)), (\gamma, \tau(i))\}.$$

Fair Lattice

Sketch Proof:

1. Pick two fair allocations.
2. Define Transfer graph.
3. Implement weakly welfare-increasing cycles.
4. (Key) Make sure you maintain feasibility.

Fair Lattice



Implementation

Algorithm to compute EBFE

1. Descending “auction” to compute distribution.
2. DA for schools at capacity.

Large market analysis:

Theorem 3 The student-optimal EBFE mechanism is asymptotically strategyproof.

Generic Domain

Theorem 4 On the NCBI and No Overlapping Cycles domain, there exists a student-optimal EBF.

Conclusion

Thank you!

School Finance

Sorting & Peer Effects

School Choice

Literature

School Choice

Abdulkadiroğlu and Sönmez [2003], Phan et al. [2024].

Matching with Externalities

Sasaki and Toda [1996], Hafalir [2008], Bando [2012], Bando [2014], Dutta and Massó [1997], Echenique and Yenmez [2007], Roth [1984], Aldershof and Carducci [1996], Roth and Peranson [1999], Klaus and Klijn [2005], Kojima et al. [2013], Ashlagi et al. [2014], Dur and Wiseman [2019], Dur et al. [2022], Pycia [2012], Rostek and Yoder [2020], Pycia and Yenmez [2023].

Literature

Price Equilibrium with Rigidities

Drèze [1975], Talman and Yang [2008], Andersson and Svensson [2014], Herings [2018].

Price As Cutoff Vector

Balinski and Sönmez [1999], Sönmez and Ünver [2010], Azevedo and Leshno [2016], Dur and Morrill [2018].

Crowding

Tierney [2019].

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